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THE PROBLEM OF FUGITIVE INTERCEPTION ON A PLANE IN THE ONE-DIMENSIONAL VECTOR FIELD OF A MOVING FLUID

The process of pursuit on the horizontal plane of a fugitive boat V by a pursuing boat U in a one-dimensional field of moving fluid is considered. The subject of the research is a mathematical model and paradoxical characteristics and properties of the persecution process. The main goal of the study was to establish the trajectory of the interception of the fleeing boat by the pursuit boat in the moving fluid flow.

In the work, a mathematical model of the process of chasing a fugitive boat on a plane is built in the form of four normal differential equations of the first order. The model takes into account the speed of the river, which forms a one-dimensional field of moving fluid. It is assumed that the pursuer U at each moment of time tries to move along the instantaneous line that visually connects two current points (him and the fugitive) on the horizontal plane, that is, the pursuer "keeps the course on the fugitive" during the pursuit. At the same time, the fugitive V tries to cross the river as quickly as possible, moving in the direction of the "life line". The pursuer tries to catch up with the fleeing boat before it reaches a certain point, which is on the "lifeline".

The method of numerical solution and research of the resulting system of differential equations was proposed and implemented in the MathCad package, and the equation of the pursuit curve was found using it. The main characteristics of the pursuit curve were established and analyzed, certain paradoxical phenomena were noted. Among the main characteristics of the pursuit process, the dependence between the amount of horizontal movement of the fugitive V towards the "lifeline" and the angular coefficient of his escape line in stagnant water was obtained. On the graph of this dependence, a well-defined local maximum is observed, which is reached at some non-zero value of the angular coefficient k. Numerical analysis also showed that there is some minimum limit of the ratio α_{MIN} of the speed modules of the pursuer U and the fugitive V, for which the pursuer U will be able to detain the fugitive within the given lane $x \in [0, X]$ before the fugitive crosses the "life line". If the value α is less than the specified value α_{MIN}, then the fugitive V will not be detained in this lane.

Key words: *the pursuer, the fugitive, the process of pursuit, pursuit curve, "life line", escape line, successful escape, the speed of the river.*

Formulation of the problem. Pursuit problems have always been among those that have interested various researchers since ancient times [1–4]. With the development of technical means and with the growth of computing capabilities and capacities of modern computers, the pursuit tasks became more precise and complicated, especially in the military field of interception of combat targets [5–22]. At the same time, in the majority of formulations of such problems, the process of pursuit was considered in stationary conditions of the external environment, that is, it did not depend on the change in the position in time of the points of space in which both the pursuer and the fugitive are instantly located. This work considers just such a case.

The object of the study is the process of chasing and intercepting a fugitive boat by a chasing boat on the horizontal surface of the river, taking into account the speed of its current.

The subject of the study was the mathematical model and paradoxical properties and characteristics of the specified persecution process.

The purpose of the research: to find 1) the trajectory of the interception L of the fugitive boat V by the pursuit boat U in moving water; 2) the point on the surface of the river in which the pursuer will catch up with the fugitive; 3) the angular coefficient k of the escape line, which ensures the maximum movement $x(k)$ of the escapee along the horizontal axis perpendicular to the "life line".

So, consider the starting position of two moving points, one of which is chasing the other (Fig. 1).

On one bank (indicated in Fig. 1 as the ordinate axis OY) of a raging river, there are two boats at a distance y_0 from each other – a coast guard boat U (hereinafter the pursuit boat) and a border violator boat V (hereinafter the fugitive boat). The river separates

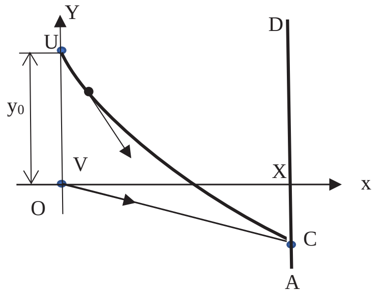

Fig. 1. Pursuit curve

the two countries as a natural border between them. At the same time, the runaway boat tries to cross the river as quickly as possible, moving in the direction of the border AD between the two countries. Let's call it “lifeline” in the future. We believe that the “lifeline” AD for the escape boat is parallel to the initial shoreline OY of the two boats. The chasing boat U tries to catch up with the boat V before it has time to reach a certain point, which is on the “lifeline”. Point C in Fig. 1 is the point of possible detention of the fugitive V by the pursuer U.

So, at the initial time $t = 0$, the runaway boat V (relative to the moving river) begins to move along a straight line $y = kx$ with an angular coefficient k (Fig. 1) and with a speed whose modulus is constant $\gamma_V = \sqrt{p^2 + q^2} = const$. Here, the values p and q are the horizontal and vertical projections of the speed of the boat V, respectively, which it can develop as much as possible in stagnant water. The pursuing boat U starts at the same time as the fleeing boat V and in the process of pursuit chooses the direction of movement to the current location of the boat V on the plane OXY. At the same time, the boat U also has a constant speed module $\gamma_U = \sqrt{u^2 + v^2} = const$, which is α times greater than the speed module of the boat V: $\gamma_U = \alpha\gamma_V$. Here, the variables u and v are the horizontal and vertical projections of the speed γ_U of the boat U, respectively, which it can develop as much as possible in stagnant water. Constant values γ_U and γ_V in a certain way depend on the power of the engines of both boats.

As it was said above, the specified speed modules of both boats are set relative to the stationary surface of the water without taking into account the transferable speed of the river current. But in this problem, the movement of boats is considered in moving water, and the module of the speed of the river current is given by the function $f(x)$ of the horizontal coordinate x, and the velocity vector is directed in the opposite direction in relation to the positive direction of the ordinate axis OY.

Physically, the movement of both boats can be identified with the movement of material points in the horizontal vector field $\vec{B}$ of velocities of a moving fluid. In this case, the components of the vector field of the moving fluid have the form: $\vec{B} = (0, -f(x), 0)$. Vertical kinematics of boat movement (that is, movement in the direction of the applique axis) is not considered in this problem.

Analysis of recent research and publications. The pursuit curve is the line along which the pursuer moves while pursuing the fugitive. The problem of persecution probably dates back to the time of Leonardo da Vinci. He was the first to investigate this problem when the fugitive moved in a horizontal straight line. The general case was studied by the French scientist Pierre Bouguer in 1732 [1–4]. The task was to find the curve of pursuit of a merchant ship by a pirate ship. At the same time, it was assumed that the speeds of the two vessels are always in the same ratio.

In works [4–7], the well-known problem of “chasing four mice” is considered. Suppose four mice are located in each of the four corners of a square table, and each mouse runs to the one to the right of it. We need to find parametric curves that describe the trajectory of each mouse. The solutions of this problem will be spiral trajectories that coincide in the center of the table.

In [8], deterministic continuous pursuit is considered, in which n ants chase each other in a circle and have predetermined variable speeds. Two discrete analogues are considered, in which a cricket or a frog is cyclically pursued with a constant and equal speed. The possible evolution of this motion as time approaches infinity is explored: collisions, limit points, equilibrium states, and periodic motion.

The paper [9] presents a simple mathematical model of the local interaction of a colony of ants or other natural or artificial creatures with a great “sense of global geometry” to find a direct path from the anthill to food. This task was also considered within the framework of the general task of chasing n ants.

The paper [10] investigates the movement of an arbitrary set of points (or beetles) on a plane chasing each other in cyclic pursuit. It is shown that for regular centrally symmetric configurations, analytical solutions are easily obtained by switching to the corresponding rotating frame of reference. Several cases of asymmetric configurations are discussed. In particular, it is shown that for three beetles in a triangular configuration of a centrally rotating coordinate system, relative to which the beetles have no tangential velocity, is the point of collapse and coincides with one of the two Brocard points in the triangle. For the case where all beetles have the same speed, a theorem is proved that whenever premature (ie, non-reciprocal) capture occurs, the collision must be head-on. This theorem is then applied to the case of three- and four-beetle configurations to show that these systems collapse up to a point, i.e., the capture is mutual. Some aspects of these results are generalized to the case of systems with n beetles.

The paper [11] investigates the motion of an arbitrary set of points on a plane chasing each other, moving in a circle. A set of marked points 1, 2, 3,..., n with coordinates (x_1, y_1), (x_2, y_2),…, (x_n, y_n), is given, and the point (x_i, y_i) is considered to be the chaser point of the fugitive point (x_{i+1}, y_{i+1}) along the instantaneous line that connects. To close the loop, the point (x_n, y_n) must follow the point (x_1, y_1). All points move at the same speed. Motion described in this way is called motion in "direct time." If the sign of the time derivative operator is changed, causing the point (x_i, y_i) to move away from the point (x_{i+1}, y_{i+1}), the motion is called "reverse-time" motion. Motion in both forward and reverse time is studied using a computer program. In real time, it is observed that eventually, in almost all tested cases, the points become collinear, with a large ratio of maximum to minimum distance, regardless of the shape of the initial configuration. In reverse time, it is observed that the points eventually arrange themselves in a star pattern, with the angle of the star apex less than some number in the neighborhood of ninety degrees. An analysis based on the stability of the equations of motion calculates the exact value of the maximum star angle as a function of the number of points.

The work [12] considers a well-known problem that continues to be analyzed today. Three (or more) beetles, which are initially located at the vertices of a regular polygon, begin to cyclically chase each other, moving together at the same speed. It is usually necessary to know the distance traveled by each beetle before mutual capture. The case of four beetles is the simplest. Due to symmetry, the four beetles always remain on the four vertices of the square. Since the fleeing beetle B2 always moves at right angles to the pursuing beetle B1, the capture speed of these two beetles depends only on the speed of the beetle B1. The case for n beetles is only slightly more complicated. The problem is again symmetrical, since n beetles are always located in n vertices of a regular n-gon.

In the book [13], systematic methods of winning in differential pursuit and evasion games are presented, the scope and application of game procedures are investigated. Numerous examples are helpful and illustrate basic and advanced concepts, including capture, strategy selection, and algebraic theory. In addition, the book contains a review of both linear and non-linear games. Chapters in the book also include stroboscopic and isochronous target acquisition.

In [14], the formulation of the pursuit problem on the plane from the point of view of the multi-agent approach is proposed. Disagreements are shown that distinguish the proposed formulation of the problem from the formulation of such a problem from the point of view of the theory of differential games. A list of methods to be developed is given.

The article [15] considers differential pursuit games on the plane, in which a group of pursuers is created for each of the fugitives. Formulated tasks of optimization of pursuit groups. Numerical methods for solving such problems were constructed, numerical experiments were conducted, and the effectiveness of these methods was analyzed.

The article [16] considers the pursuit problem with simple movement for the case when the maximum speeds of the players are the same, and the fleeing person moves along a strictly convex smooth n-dimensional surface. It is proven that the end of pursuit is possible from any starting position. It is established that evasion is possible from some initial positions if the hypersurface contains a two-dimensional flat part.

In the book [17], problems from the classical theory of optimal control are considered, in which optimal control is determined, which optimizes a criterion subject to a dynamic constraint, which expresses the evolution of the system state under the influence of control variables. Different differential games are studied when extended to the case of multiple controllers (or players) with different and sometimes conflicting optimization criteria (gain function). The most developed part of differential games are zero-sum differential games (or differential pursuit games), which are extensively studied. The book develops a complete theory of differential pursuit games with full and partial information. Numerous specific pursuit-evasion games (lifeline games, simple pursuit games, etc.) are solved, as well as new time-consistent optimality principles in n-person differential game theory.

In the book [18], differential games are considered as conflict situations with an infinite set of alternatives that can be described using differential equations. In the book, the main attention is paid to fundamental mathematical questions, the solution methods are illustrated by a large number of interesting examples, which are important in themselves. A number of unsolved problems for independent work are offered.

In the message [19], the persecution process was analyzed from a military point of view. As pursuers can be considered: missiles, interceptor aircraft (or other aircraft); intelligent search for a military target (for example, ammunition depots, tanks, aircraft at airfields, etc.); a unit or individual soldier pursuing and approaching an enemy unit or individual soldier; ships approaching other ships, tracking torpedoes that explode when they hit enemy ships. In this paper, the pursuit process was considered under the following assumptions: the pursuer moves at a constant speed, the pursuit takes place in a given direction during a certain time interval, and the pursuer always sees the fugitive (target). The simulation of the pursuit process was first performed for the flat case, and later this model was generalized to the three-dimensional case. The purpose of the study was to determine the movement curve for the pursuer, provided the location of the target is known.

The technical report [20] provides a detailed description of the implementation of a clean pursuit curve tracking algorithm. Given the general success of the algorithm in recent years, it is likely that it will be used again in land navigation tasks. The report also includes a geometric version of the method and provides some information on the performance of the algorithm as a function of its parameters.

The article [21] describes a third-order chase, a game of evasion in which both players have the same speed and minimum turning radius. A idiosyncratic game is first resolved for a barrier or shell of states that can be captured. When capture is possible, the game at some level is decided for optimal control of the two players as a function of relative position. It is found that the solution of the problem includes a universal surface for the pursuer and an acceleration surface for the evaders.

The article [22] considers the coplanar problem of evading pursuit, in which two pursuers P_1 and P_2 and one fugitive E participate. The fugitive E evades with a constant speed w > 1 greater than that of the pursuers, and must pass between the two pursuers P_1 and P_2 with a single speed, the gain is the distance of closest approach to any of the pursuers. This is a typical two-player zero-sum game theory problem. Velocity directions P_1, P_2 and E are chosen as controlling variables. A closed-loop solution is obtained through elliptic functions of the first and second kind. The closed-loop solution is shown graphically in several diagrams for different values of w.

The considered problem is a continuation of previous studies [23–25], which considered the problem of the fastest movement of a boat in moving water in a variation setting.

Construction of the differential equation of the trajectory of the pursuit boat.

Let us identify the location of the boats on the plane OXY with the points U and V. We denote by (x, y) the coordinates of a point U in moving water and by (ξ, η) – the coordinates of a point V in moving water at the current moment of time t. Let's build a system of differential equations to establish the pursuit curve taking into account the transferable speed of the river current:

$$\begin{cases} \dfrac{dx}{dt} = u; \\ \dfrac{dy}{dt} = v - f(x); \\ \dfrac{d\xi}{dt} = p; \\ \dfrac{d\eta}{dt} = q - f(x); \end{cases} \tag{1}$$

Let's set the expression for the angular coefficient y' of the tangent to the curve L at the current point $U(x, y)$:

$$y' = (v - f(x))/u\,. \tag{2}$$

Note that the pursuer at each moment of time tries to move along the instantaneous line that visually connects the two current points $U(x, y)$ and $V(\xi, \eta)$ on the plane OXY (that is, the pursuer $U(x, y)$ "keeps track of the fugitive" $V(\xi, \eta)$). At the same time, this direction must coincide with the direction of the tangent y' to the desired pursuit curve $U(x, y)$. Let's describe it mathematically, taking into account (2). As a result, we will get the equality of two angular coefficients:

$$(v - f(x))/u = (\eta - y)/(\xi - x)\,. \tag{3}$$

First, let's get rid of the variables u and v in system (1) using equality (3) and the relation for γ_U. To do this, let's express the variable v in terms of the remaining variables (x, y, ξ, η) from equation (3).

$$v = f(x) + u \cdot \omega(x, y, \xi, \eta)\,, \tag{4}$$

where $\omega(x, y, \xi, \eta) = (\eta - y)/(\xi - x)$. In further transformations, the arguments of the function $\omega = \omega(x, y, \xi, \eta)$ will be omitted. Now let's use relations $\gamma_U = \sqrt{u^2 + v^2}$ and expression (4) to get rid of the variable u. As a result, we get a quadratic equation

$$u^2 \cdot (1 + \omega^2) + 2f(x)\omega u + f^2(x) - \gamma_U^2 = 0\,. \tag{5}$$

Solutions of equation (5) are a pair of relations:

$$\tilde{u}_{1,2} = \left[-f(x)\omega \pm \sqrt{\gamma_U^2(1 + \omega^2) - f^2(x)}\right] \Big/ (1 + \omega^2)\,. \tag{6}$$

The corresponding two solutions for v have the form:

$$\tilde{v}_{1,2} = \pm\sqrt{\gamma_U^2 - u_{1,2}^2}\,. \tag{7}$$

Now, in system (1), let's get rid of the variables p and q using the relations $\gamma_V = \sqrt{p^2 + q^2}$ and $q = kp$. In this problem, these variables are constant and have the following form:

$$\tilde{p}_{1,2} = \pm\gamma_V \Big/ \sqrt{1 + k^2}\,,\ \tilde{q}_{1,2} = \pm\gamma_V k \Big/ \sqrt{1 + k^2}\,. \tag{8}$$

Substituting the obtained expressions (6) – (8) into system (1), we obtain a normal system of four nonlinear differential equations that describes the process of chasing a fugitive in moving water:

$$\begin{cases} \frac{dx}{dt} = \tilde{u}(x, y, \xi, \eta); \\ \frac{dy}{dt} = \tilde{v}(x, y, \xi, \eta) - f(x); \\ \frac{d\xi}{dt} = \tilde{p}; \\ \frac{d\eta}{dt} = \tilde{q} - f(x); \end{cases} \qquad (9)$$

Numerical analysis of the process of pursuit of a fugitive in moving water.

For the numerical integration of system (9), the following initial conditions are chosen: $x(0) = \xi(0) = \eta(0) = 0; y(0) = 2$. So, we have the following starting points: for the pursuing boat it is point $U(0, 2)$, and for the runaway boat – $V(0, 0)$ respectively. The function , which sets the modulus of the river current, has the form:

$$f(x) = A_0 \sin(\pi x / H). \qquad (10)$$

We present a graph (Fig. 2) constructed for the case of the pursuit process, in which the following movement parameters are selected: $k = -1$, $A_0 = 1$, $H = 2$ and $\gamma_U / \gamma_V = \alpha = 2$. On the graph in Fig. 2, the dashed curve represents the trajectory of the runaway boat V, and the solid curve represents the trajectory of the pursuing boat U. Note that the dashed curve in moving water is not straight, but has a certain curvature due to the transfer speed of the river current.

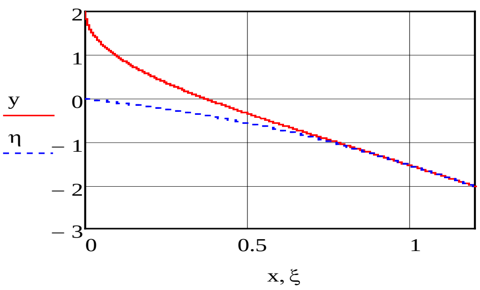

Fig. 2. Pursuit (solid curve) and escape (dashed curve) curves for the case: k=–1, A_0=1, H=2, α=2, y_0=1

The point on the plane OXY where the fugitive will be apprehended has coordinates: $C(1.264, -2.170)$. The time T that will be spent on the pursuit process in this case is equal to: $T = 1.787$ (time units).

Now let's examine the behavior of the function $x(k)$ (this is the horizontal coordinate of the fugitive's arrest point) depending on the variable angular coefficient k of the straight line along which the fugitive would move in stagnant water. For this, the following fixed parameters were chosen: $A_0 = 1$, $H = 1$, $y_0 = 1$ and $\alpha = 2$. The following initial conditions are chosen for integration: $x(0) = \xi(0) = \eta(0) = 0; y(0) = 1$. Thus, $U(0, 1)$ – the starting point of the pursuit boat; $V(0, 0)$ – the starting point of the runaway boat's movement. Fig. 3 shows the indicated graph, on which a clearly expressed local maximum is observed, which is reached at a certain value of the angular coefficient k. In our case, it is approximately equal to $k = -0,450$.

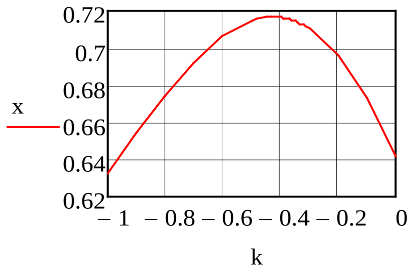

Fig. 3. Graph of dependence of the maximum horizontal movement of the fugitive towards the "life line"

The local maximum of the curve indicates that there is a certain non-zero angle of inclination of the escape boat's direct motion (in stagnant water), which gives it the opportunity to achieve the maximum movement towards the "lifeline" AD. So, we observe a certain paradox: the curve in Fig. 3 establishes that the strategy of choosing an angle of inclination equal to zero for direct escape in standing water (that is, movement along the shortest segment connecting two parallel lines OY and AD – which would be natural) is not correct for a successful escape. In this version of the calculation ($\alpha = 2$, $k = -0,450$), this movement is as follows: $x(-0,45) = 0,717$ (distance units). For comparison, we note that if the "life line" AD were, for example, at a distance of $X = 0,700$ (units of distance) from the ordinate axis, then the escape of the boat V would be successful, unlike the case when a straight line with a zero angle coefficient was chosen for escape (i.e. abscissa axis). In the latter case $x(0) = 0.642 < X$, and the fugitive will definitely be detained.

Numerical analysis of the pursuit process also showed that there is a certain minimum limit of the ratio of speed modules $\gamma_U / \gamma_V = \alpha_{MIN}$, at which the pursuer U will be able to detain the fugitive V within the given lane $0 < x < X$ before the fugitive crosses the "life line" AD. If the value α is less than the specified value α_{MIN}, then the fugitive V will not be apprehended in this lane. Recall that the ratio α establishes a connection between the power of the motors of the two boats. Moreover, the value α_{MIN} is significantly different from the similar value in the case when the speed of the river flow is not taken into account in the formulation of the problem.

To illustrate this interesting property, consider the process of chasing a fugitive in the case of $k = -0,450$, $A_0 = 1$, $H = 1$, $y_0 = 1$, $X = 1,00$. First, we will find the minimum value α_{MIN} that will ensure the arrest of the fugitive within the given lane $0 < x < 1,00$. A numerical experiment showed that it is equal to: $\alpha_{MIN} = 1,69$. Fig. 4 shows the pursuit curve constructed for the considered case.

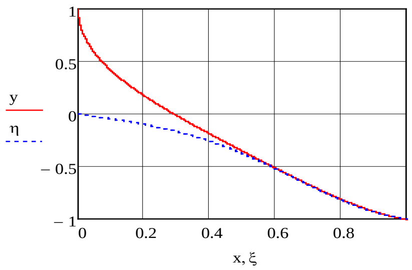

Fig. 4. The process of chasing and apprehending a fugitive in a given lane $0 < x < 1,00$ at a speed ratio $\alpha_{MIN} = 1,69$

The intersection of the two curves occurs at the point $C(0,997;-1,001)$, which indicates that the fugitive V was apprehended within the given lane $0 < x < 1,00$.

Conclusions. The work solves the actual problem of creating a mathematical model of the process of chasing and apprehending a fugitive in a horizontal vector field of a moving fluid with certain restrictions on the movement of the fugitive.

The scientific novelty of the obtained results lies in the fact that for the first time a mathematical model of the process of chasing a fugitive was formulated and built, taking into account the transferable speed of the external environment.

The practical significance of the obtained results is that based on the developed mathematical model and its software implementation, a numerical analysis of the fugitive pursuit process was carried out and its important characteristics and properties were established. One of them concerns the choice of the direction of escape and indicates that there is a certain non-zero angle of inclination of the rectilinear (in still water) movement of the fleeing boat, which, out of all other possible angles, makes it possible to achieve its maximum movement closest to the “lifeline”. Thus, this paradoxical property establishes that the choice of the angle of inclination of the escape line, which is equal to zero, is not optimal for a successful escape. The results of the numerical experiment allow recommending the developed mathematical model and its software implementation for further practical use in the study of pursuit processes in horizontal vector fields of a moving fluid.

Prospects for further research will be related to the study of the features and properties of the pursuit process for the case when the fugitive chooses an algebraic curve of the second order (in a standing fluid) for escape, provided that the transfer velocity of the moving fluid is taken into account.

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Легеза В.П., Нещадим О.М. ЗАДАЧА ПЕРЕХОПЛЕННЯ ВТІКАЧА НА ПЛОЩИНІ В ОДНОВИМІРНОМУ ВЕКТОРНОМУ ПОЛІ РУХОМОЇ РІДИНИ

Розглядається процес переслідування на горизонтальній площині катера-втікача V катером-переслідувачем U в одновимірному полі рухомої рідини. Предметом дослідження є математична модель та характеристики і властивості процесу переслідування. Основною метою дослідження було встановлення траєкторії перехоплення катера-втікача V катером переслідування U у рухомому потоці рідини. Побудовано математичну модель процесу переслідування катера-втікача V на площині у вигляді чотирьох нормальних диференціальних рівнянь першого порядку. Модель враховує швидкість течії річки, яка утворює одновимірне поле рухомої рідини. Припускалося, що переслідувач U в кожен момент часу намагається рухатися по миттєвій лінії, яка візуально з'єднує дві поточні точки (його і втікача V) на горизонтальній площині, тобто переслідувач U «тримає курс на втікача» під час переслідування. При цьому втікач V намагається якомога швидше перетнути річку, рухаючись у напрямку «лінії життя» AD. В свою чергу, переслідувач U намагається наздогнати катер-втікач V до того, як той встигне досягти певної точки, яка знаходиться на «лінії життя» AD. Запропоновано і реалізовано в пакеті MathCad метод числового розв'язання і дослідження отриманої системи диференціальних рівнянь та з його використанням знайдено рівняння кривої переслідування. Встановлено і проаналізовано основні характеристики кривої переслідування, відмічені певні парадоксальні властивості. Серед основних характеристик процесу переслідування отримано залежність між величиною горизонтального переміщення втікача V у бік «лінії життя» та кутовим коефіцієнтом k його лінії втечі в стоячій воді. На графіку цієї залежності спостерігається чітко виражений локальний максимум, який досягається при деякому ненульовому значенні кутового коефіцієнта k. Числовий аналіз також показав, що існує деяка мінімальна межа α_{MIN} співвідношення модулів швидкостей переслідувача U і втікача V, за якого переслідувач U зможе затримати втікача V в межах заданої смуги $x \in [0, X]$ до того, як втікач перетне «лінію життя». Якщо значення $\alpha < \alpha_{MIN}$, то втікач V не буде затриманий на даній смузі.

Ключові слова: *втікач, переслідувач, процес переслідування, крива переслідування, «лінія життя», лінія втечі, успішна втеча, швидкість течії річки.*